## Vasicek model

1.In this problem,you are asked to price an interest rate swap and calculate its Value-at-Risk .This swap is a contract that exchange floating rates payments for fixed rates payments between two parties every quarter.The setting for this interest rate swap is as allows

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| --- | --- |
| Today: | 2020/11/27 |
| Maturity Date: | 2025/11/26 |
| Payment Frequency: | half a year |
| Face Value: | le6 |
| Fixed Rate: | 3.582% |

Assume that the instantaneous interest rate is captured by the Vasicek Model.The Vasicek model of interest rates specifies the risk-natural(physi- cal)dynamics for the instantaneous interest rates

drt=m(rt,t)dt+s(rt,t)dX (1)

where m(r,t)=γ(r-rt)and s(r,t)=σ.The instantaneous interest rate

is the interest rate agents could borrow or lend between date t and t+ dt.Equation ①is known as the risk-natural (physical)dynamics for the instantaneous interest rates.

The risk-neutral dynamics for the instantaneous interest rates follows

*drt=γ\*(r\*-rt)dt+σdX* (2)

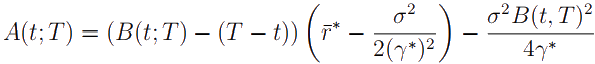
Let Z(rt,t,T)denote the time-t price for a zero coupon bond (ZCB)that matures at time T with the current instantaneous interest rate rt.The ZCB has terminal payoff

*Z(rr,T,T)=1*

The solution to the fundamental bond pricing equation yields

*Z(r,t;T)=eA(t,T)-B(t,T)r*

B(t,T)=r\*-¹(1-e-r"(T-t))



where T=T-t is the time to maturity.The excel spreadsheet China Treasury Spot Yields contains daily data on continuously compounded annualized interest rates from 2015.11.27 to 2020.11.26.Here,we use Shibor (Shanghai Interbank Offered Rate)as the floating rate.

(a)(10pts)Estimate the risk-natural parameters of Vasicek model,i.e.,y,

r and σ .

(b)(10pts)Estimate the risk-neutral parameters of Vasicek model,i.e.γ\*, F\*by non-linear least squares.The goal is to minimize the sum of squared pricing errors,that is



and set o to be the same as the estimated risk-natural o.

(c)(20pts)Price the interest rate swap by the Monte Carlo Simulation method (set seed as 'default').This interest rate swap give you fixed

rate payments but costs you Shibor 1 month rate.We make the sim- plifying assumption that the spread between Shibor 1 month rate and treasury 1 month rate is constant and equal to 0.8750bp (One basis point equals to the one hundredth of a percentage point).This means that each quarterly the payment is

*Cashflowt=FaceValue\*(FixedRate-Spread-TreasurylMonthRate)/4*

And you are required to write a pricing function with following inputs:

i.initial interest rate of treasury rate; ii.fixed rate;

iii.face value;

iv.risk-neutral parameters.

(d)Value at Risk:Use Monte Carlo Simulation and Vasicek model to compute a 0.5-year Value-at-Risk,that is the maximum loss that the portfolio may incur with α probability due to the movement in interest rates.Compute the 1%and 5%worst casesof the interest rate swap distribution,and thus obtain the 1%VaR and 5%VaR.